RADIATION PROJECT INTERNAL REPORT NO. 3

# Stored-Energy Linac Parameters

by

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These are single-particle, paraxial, completely relativistic calculations intended to establish the approximate dependence of the parameters of a stored-energy linac on energy, current, etc. Where applicable the notation is that of RPPR No. 2[1].

Consider a single $TM_{010}$ cavity. Let $\phi_c$ be the crossing phase of the electron at the center plane, $\phi_e$ be the electrical length of the cavity. Then the energy gain of the electron in crossing the cavity is

$$\Delta T = e\int E dz = (eE_o c/\omega) \int_{\phi_c - \frac{1}{2}\phi_e}^{\phi_c + \frac{1}{2}\phi_e} \cos\phi d\phi$$

$$= (2eE_o c/\omega) \cos\phi_c \sin\tfrac{1}{2}\phi_e .$$

If the cavity length is $L = \phi_e \lambda/2\pi$, then the mean energy gain per unit length is $\Delta T/L \equiv T'$, so that

$$T' = eE_o \cos\phi_c \;(\sin\tfrac{1}{2}\phi_e/\tfrac{1}{2}\phi_e). \qquad (1)$$

We adopt the Kilpatrick criterion[2] with some simplifications. We first suppose that $V/V^*$ is always small (if the ejected particle is taken to be a proton, then $V^*$ is given by

$$V^* \cong (1/\pi) \times 10^9 (L/\lambda)^2 \text{ ev});$$

it then follows from the curve in ref. 2 relating the quantities W and V that $W/V \sim 0.6\, V/V^*$, or $W \sim 0.6\, V^2/V^*$. It is recognized that by optimizing the pressure and conditioning the electrodes the Kilpatrick breakdown voltage can be exceeded by a large factor, and this voltage will therefore be used here only for reference purposes. The W-E curve of ref. 1 can, over the interesting range of parameters, be fitted by the approximate formula

$$W = 10^{20.8} E^{-3.08}$$

(units are volts and cm); it may be noted that this gives, under dc conditions, a dependence

$$W \propto L^{0.78},$$

this being reasonably near Maitland's[3] index of 0.7 and Staprans's[4] of 0.8 for vacuum tubes. Thus

$$0.6 \, (V/L)^2 \lambda^2 \, (\pi/10^9) = 10^{20.8} E_o^{-3.08}$$

for the maximum field. Putting $V/L = E_o$ and solving gives

$$E_o = 1.1 \times 10^6 \lambda^{-0.394}. \qquad (2)$$

For the stored-energy criterion we have from ref. 1

$$\lambda = 6.8 \times 10^6 \; (k/h) i t_o / T'. \qquad (3)$$

For a $TM_{010}$ cavity we have[6] (d ≡ skin depth)

$$Q = (\lambda/d) \times 0.77/(2 + 0.77\lambda/L). \qquad (4)$$

For the power per cm we have

$$P = \omega k i t_o T'/Q$$

and for the total power ($T_o$ ≡ overall energy gain)

$$P_t = PT_o/T' = \omega k i t_o T_o/Q. \qquad (5)$$

Define the constants C:

$C_1 \equiv \cos\phi_c$ (taken as 1 in the rest of these calculations)

$C_2 \equiv 1.1 \times 10^6$ (at $V=V_K$, the Kilpatrick voltage; $V/V_K$ will subsequently be made variable)

$C_3 \equiv 6.8 \times 10^6 \; (k/h) i t_o$

$C_4 \equiv 4.1 \times 10^3$ (for copper)

$C_5 \equiv 0.413$

$C_6 \equiv 18.8 \times 10^{10} k i t_o T_o$

$C_7 \equiv C_3{}^2/C_1 C_2 = 42 \times 10^6 \; (k/h) i t_o \sec\phi_c.$

Now define normalized variables according to $T' \equiv K_1\tau$, $E_o \equiv K_2\eta$,
$\lambda \equiv K_3\mu$, $Q \equiv K_4 q$, $P_t \equiv K_5 p$, so that

$$\tau = \eta(\sin \tfrac{1}{2}\phi_e/\tfrac{1}{2}\phi_e),\ \eta = \mu^{-0.394},\ \mu = \tau^{-\frac{1}{2}},\ q = \mu^{\frac{1}{2}}/(C_5 + 1/\phi_e),\ p = 1/q\mu.$$

On comparing these with eqs. (1) through (5), one can obtain the K's in terms of the C's:

$$K_1 = C_1C_3C_7^{-0.245}$$

$$K_2 = C_3C_7^{-0.246}$$

$$K_3 = C_7^{0.6226}$$

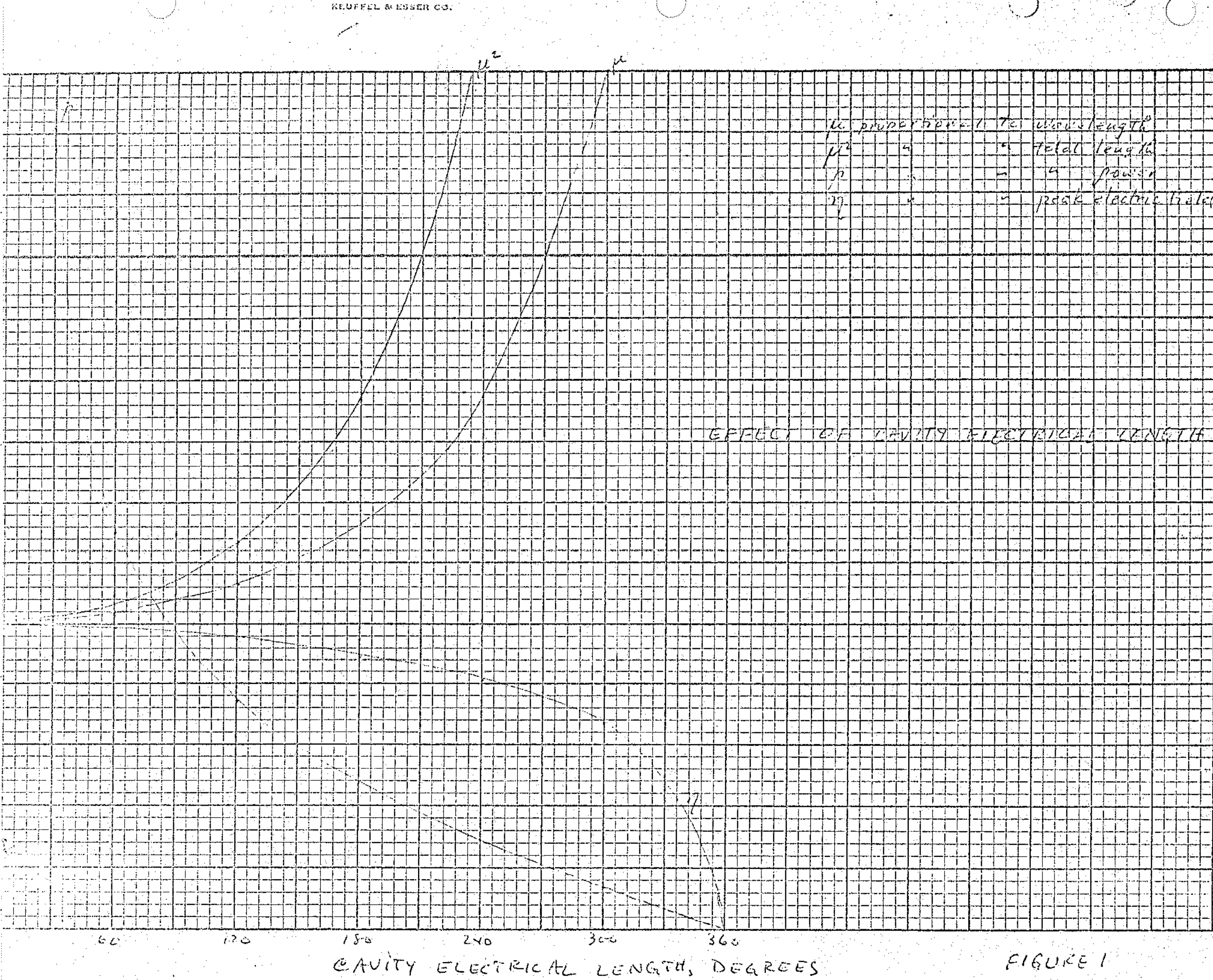
$$K_4 = C_4C_7^{0.3113}$$

$$K_5 = (C_5/C_4)C_7^{-0.9339}.$$

We reduce the normalized equations to

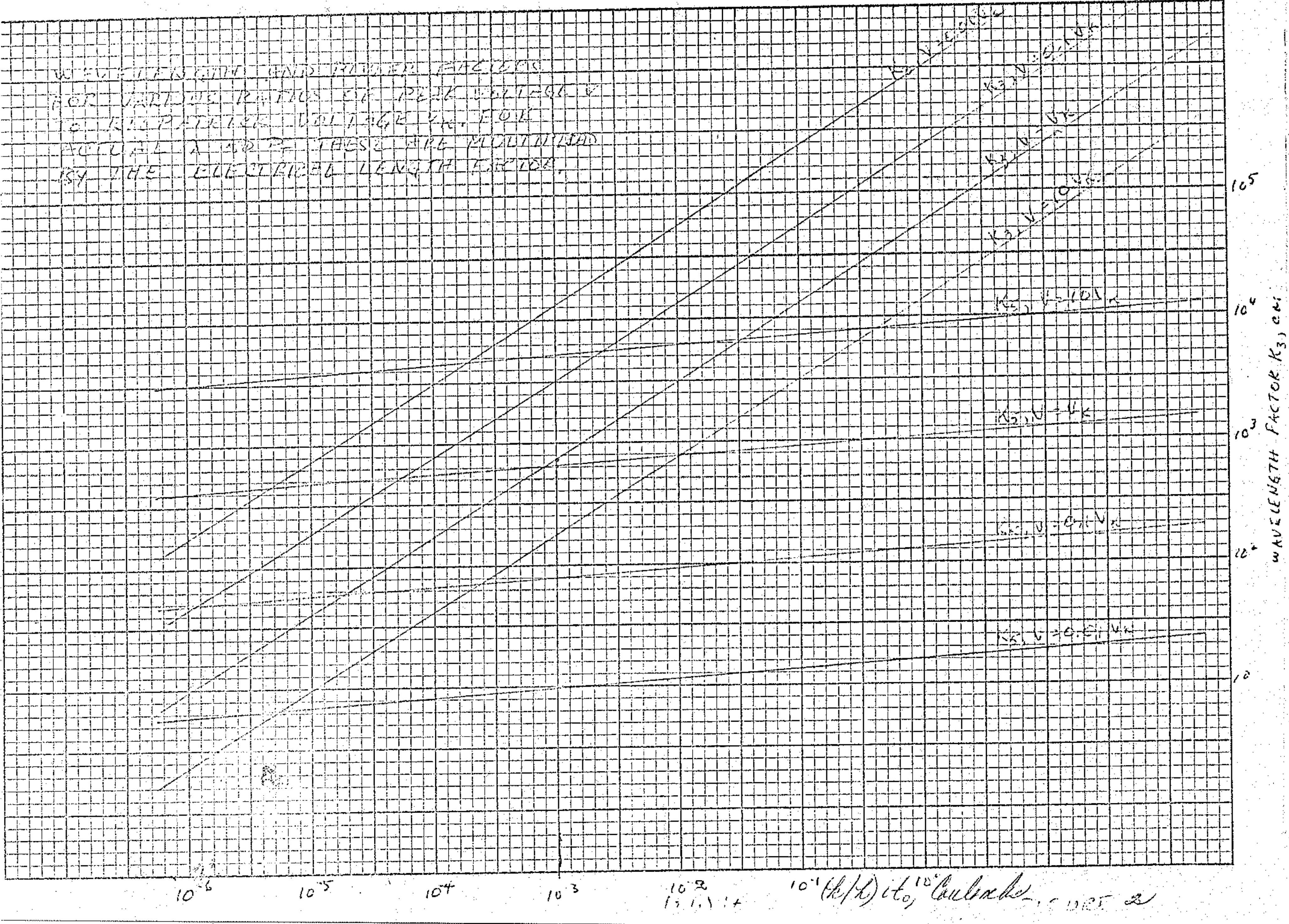
$$\mu = (\tfrac{1}{2}\phi_e/\sin \tfrac{1}{2}\phi_e)^{0.6226},\ p = \mu^{-3/2}(0.413 + 1/\phi_e);$$

these are plotted in figure 1. They indicate the dependence of normalized wavelength and power on the electrical length of the cavity. Figure 2 shows the dependence of the normalizing factors $K_3$ for the wavelength and $K_5$ for the total power. To find the actual wavelength and power required these should be multiplied by the appropriate factor dependent on cavity electrical length, as displayed in figure 1. It should be particularly noted that, for fixed $V/V_K$, shortening the accelerator increases the power, as does increasing $V/V_K$ at constant cavity electrical length.



KEUFFEL & ESSER CO.
μ²
μ
μ proportional to wavelength
μ² " " total length
p " " " power
η " " " peak electric field
EFFECT OF CAVITY ELECTRICAL LENGTH
η
60
120
180
240
300
360
CAVITY ELECTRICAL LENGTH, DEGREES


FIGURE 1


THESE ARE MULTIPLIED
BY THE ELECTRICAL LENGTH FACTOR.
$K_3$, V=10 $V_K$
$K_3$, V=$V_K$
$K_3$, V=0.1 $V_K$
WAVELENGTH FACTOR $K_3$, cm
10
$10^2$
$10^3$
$10^4$
$10^5$
$10^{-6}$
$10^{-5}$
$10^{-4}$
$10^{-3}$
$10^{-2}$
$10^{-1}$ (k/k) $i t_0$, Coulombs

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5. A. F. Harvey, "Principles of Microwave Engineering", Academic Press, 1963, p. 196.